

# MONOPOLES IN QCD

H. ICHIE and H. SUGANUMA

*Research Center for Nuclear Physics, Osaka University*

*E-mail: ichie@rcnp.osaka-u.ac.jp*

Based on the dual Higgs picture for confinement, we study monopole properties in the maximally abelian (MA) gauge using the lattice QCD. The monopole carries a large fluctuation of the gluon field and provides a large abelian action in abelian projected QCD (AP-QCD). We find large cancellation between abelian part and off-diagonal part of the action density around the monopole, which ensures the appearance of monopoles. The monopole in the MA gauge can be regarded as the collective mode of the large gluon fluctuation concentrated into the abelian sector, and would be relevant degrees of freedom for confinement in AP-QCD.

In the 't Hooft abelian gauge, QCD is reduced into an abelian gauge theory with magnetic monopoles<sup>1</sup>, and color confinement can be understood by the dual Meissner effect with monopole condensation. In relation to the dual Higgs picture, recent lattice QCD simulations show strong evidence of abelian dominance and monopole dominance for confinement in the maximally abelian (MA) gauge. Abelian projected QCD (AP-QCD) is defined by neglecting off-diagonal gluon components in the MA gauge, and it almost reproduces the essence of nonperturbative features of QCD.

We investigate the theoretical structure of AP-QCD by comparing with QCD and QED as shown in Fig.1. As for confinement properties, AP-QCD resembles the original QCD, while QED does not provide confinement phenomena. The difference between QCD and QED is existence of off-diagonal components of the gauge field, which leads to the asymptotic freedom and strong-coupling nature in the infrared region of QCD. However, AP-QCD has an abelian gauge symmetry and resembles QED rather than QCD on the symmetry. Then, why does AP-QCD provide confinement without off-diagonal gluons? The obvious difference between AP-QCD and QED is existence of monopoles. As a natural conjecture, the monopole in AP-QCD plays an alternative role of off-diagonal gluons and is responsible for confinement phenomena. Then, the monopole would be closely related to off-diagonal gluons.

Next, we consider monopole properties in terms of the action. In the static frame of the monopole with the magnetic charge  $g$ , a spherical 'magnetic field' is created around the monopole in the abelian sector of QCD as  $\mathbf{H}(r) = \frac{g}{4\pi r^3} \mathbf{r}$  with  $\mathbf{H}_i \equiv \epsilon_{ijk} \partial_j A_k^3$ . Then, the monopole inevitably accompanies a large fluctuation of the abelian gluon component  $A_\mu^3$  around it. For the abelian part  $S_{\text{Abel}} \equiv -\frac{1}{4} \int d^4x (\partial_\mu A_\nu^3 - \partial_\nu A_\mu^3)^2$  of the QCD ac-

tion, the electro-magnetic energy created around the monopole is estimated as  $E(a) = \int_a^\infty d^3x \frac{1}{2} \mathbf{H}(r)^2 = \frac{g^2}{8\pi a}$ , where  $a$  is an ultraviolet cutoff like a lattice mesh. As the “mesh”  $a$  goes to 0, the monopole provides a large fluctuation of  $S_{\text{Abel}}$ , and hence the monopole seems difficult to appear if the abelian gauge theory is controlled by  $S_{\text{Abel}}$ . This is the reason why QED does not have the point-like Dirac monopole. Then, why can the monopole appear in the abelian sector of QCD? To answer it, let us consider the division of the total QCD action  $S_{\text{QCD}}$  into the abelian part  $S_{\text{Abel}}$  and the remaining part  $S_{\text{off}} \equiv S_{\text{QCD}} - S_{\text{Abel}}$ , which is contribution from the off-diagonal gluon component. Unlike  $S_{\text{QCD}}$  and  $S_{\text{Abel}}$ ,  $S_{\text{off}}$  is not positive definite and can take a negative value in the Euclidean metric. Then, around the monopole, the abelian action  $S_{\text{Abel}}$  should be partially canceled by the remaining off-diagonal contribution  $S_{\text{off}}$  such that the total QCD action  $S_{\text{QCD}}$  around the monopole does not become extremely large. Thus, we expect large off-diagonal gluon components around the monopole for cancellation with the large field fluctuation of the abelian part. Based on this analytical consideration, we study action densities around monopoles in the MA gauge using the lattice QCD.

On the SU(2) lattice, we measure the action densities  $\bar{S}_{\text{SU}(2)}$ ,  $\bar{S}_{\text{Abel}}$  and  $\bar{S}_{\text{off}}$ , which are the SU(2), the abelian and the off-diagonal parts, respectively. We show the probability distribution of  $\bar{S}_{\text{SU}(2)}$ ,  $\bar{S}_{\text{Abel}}$  and  $\bar{S}_{\text{off}}$  in Figs.2 and 3. (To be exact,  $\bar{S}$  is the averaged value over the neighboring links around a dual link.) For total distribution on the whole lattice, most  $\bar{S}_{\text{off}}$  is positive, and both  $\bar{S}_{\text{Abel}}$  and  $\bar{S}_{\text{off}}$  tend to take smaller values than  $\bar{S}_{\text{SU}(2)}$  owing to  $\bar{S}_{\text{SU}(2)} = \bar{S}_{\text{Abel}} + \bar{S}_{\text{off}}$ . Around the monopole, however, the off-diagonal part  $\bar{S}_{\text{off}}$  of the action density tends to take a large negative value, and  $\bar{S}_{\text{off}}$  strongly cancels with the large abelian action density  $\bar{S}_{\text{Abel}}$  so as to keep the total SU(2) action small. Thus, monopoles can appear in the abelian sector in QCD without large cost of the QCD action due to large cancellation between the abelian action  $S_{\text{Abel}}$  and the off-diagonal part  $S_{\text{off}}$  of the action.

In AP-QCD, the monopole provides a large abelian field fluctuation. For confinement, this large abelian field fluctuation originated from monopoles would be important in AP-QCD in terms of the area-law reduction of the abelian Wilson loop, which leads to the abelian string tension. In conclusion, the monopole in the MA gauge can be regarded as the collective mode of the large gluon fluctuation concentrated into the abelian sector, and would be relevant degrees of freedom for confinement in AP-QCD.

## Reference

1. G. 't Hooft, Nucl. Phys. **B190** (1981) 455.

### Figure Captions

**Fig.1** Comparison among QCD, abelian projected QCD (AP-QCD) and QED in terms of the gauge symmetry and fundamental degrees of freedom.

**Fig.2** Probability distributions  $P(\bar{S})$  of the SU(2) action density  $\bar{S}_{\text{SU}(2)}$  (dashed curve), abelian action density  $\bar{S}_{\text{Abel}}$  (solid curve) and off-diagonal part  $\bar{S}_{\text{off}}$  (dotted curve) in the MA gauge at  $\beta = 2.4$  on  $16^4$  lattice.

**Fig.3** The probability distribution  $P_k(\bar{S})$  **around the monopole** for  $\bar{S}_{\text{SU}(2)}$ ,  $\bar{S}_{\text{Abel}}$  and  $\bar{S}_{\text{off}}$  in the MA gauge at  $\beta = 2.4$  on  $16^4$  lattice. The meaning of curves is same as Fig.2. Large cancellation between  $\bar{S}_{\text{Abel}}$  and  $\bar{S}_{\text{off}}$  is found.

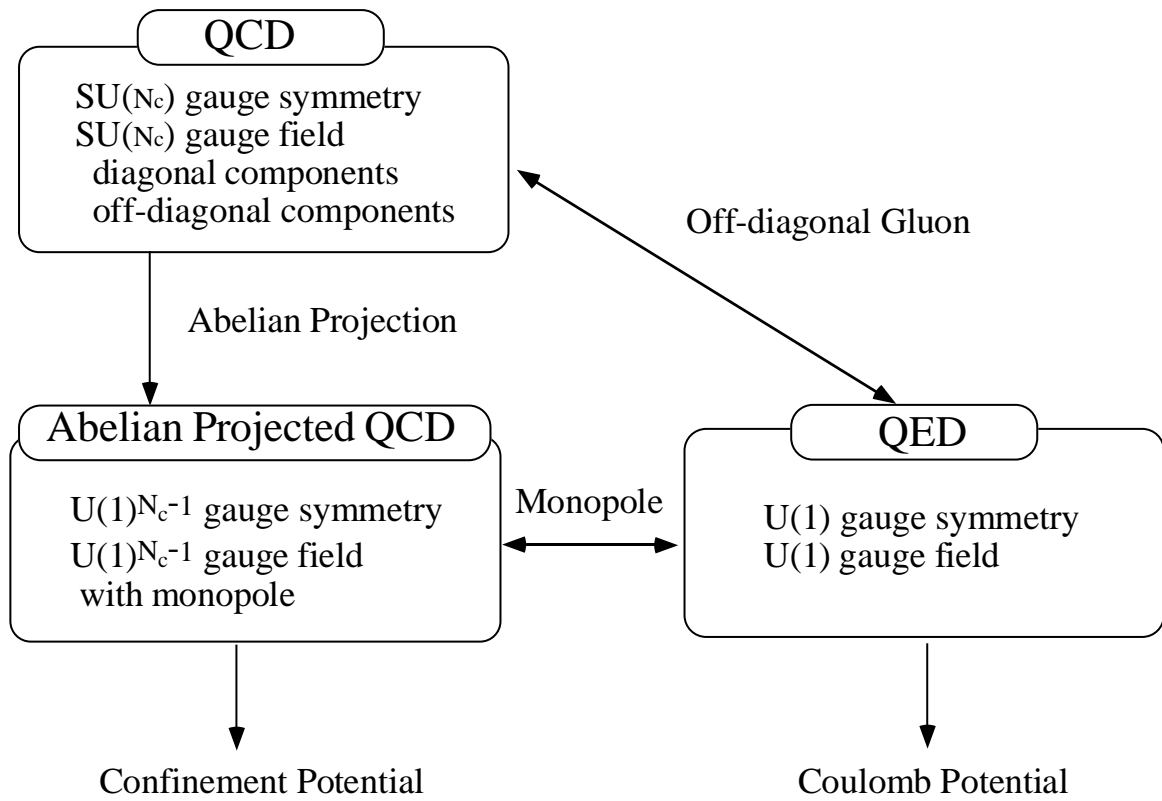


Fig.1

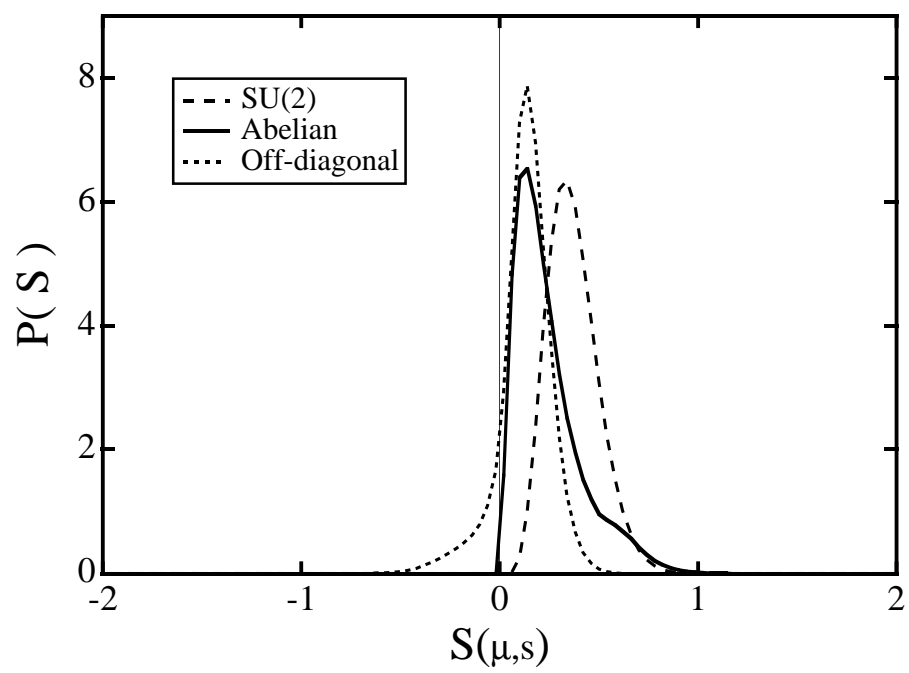


Fig.2

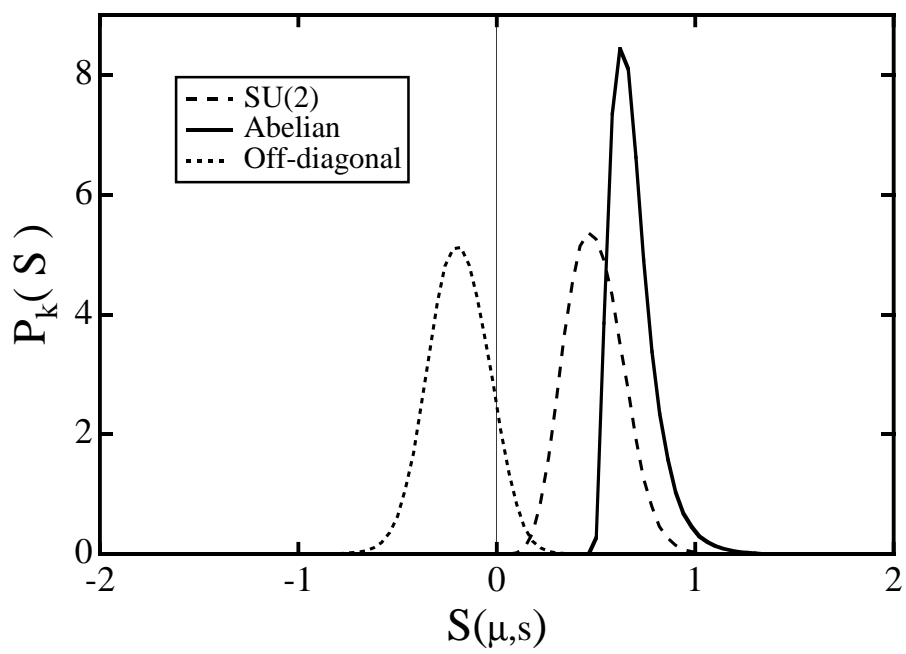


Fig.3